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Introduction

Soap films are thin layers liquid (usually water based) surrounded by air. The interface formed on merging of two soap bubbles is an example of a soap film. The film formation is not feasible with simple water. To increase the surface tension, we add detergent/soap to make a soap solution. It is due to this that the surface tension is increased enough to support the large increment in surface areas, introduced due to film formation. The film can acquire various shapes for various geometrical frames, as a result of its surface minimalisation property.

Formation

The presence of soap, which is composed at a molecular scale of surfactants, is necessary to stabilize the film. Detergents contain hydrophobic and hydrophilic parts. Thus, they are arranged preferentially at the air/water interface. Surfactants stabilize films because they create a repulsion between both surfaces of the film, preventing it from thinning and consequentially bursting. This can be shown quantitatively through calculations relating to disjoining pressure.

Minimal surface

Our universe has many 'minimal' laws.One of the prominent examples include 'the least action principle'. Similarly, there is one law which deals with the shape of all bodies in this universe formed by natural processes, which states that; everybody in this universe, under the physical influence of natural forces, tends to acquire a shape of minimum energy/surface area and maximum stability. This is what makes heavenly objects, soap bubbles and even falling water-drops round. For satisfying this, they acquire minimal surface forms. A collection of surfaces, interface and membranes is called 'minimal surface form' when it has assumed a geometric configuration of least area among those into which it can deform.

Mathematically, a minimal surface is the one, which locally minimizes the area. This equivalently means, that the mean curvature of the surface is zero. a soap film in this term is stated as:

Plateau's law

In the middle of the 19th century, physicist J Plateau proposed that minimal surfaces could be visualized using the soap film. To see the minimal surface for a given arbitrary boundary, it is sufficient to soak a wire frame with its boundary in soapy water. The soap film occupies the minimal surface to help minimize the surface tension. This demonstration, which combined mathematics and physics, was a great achievement by Plateau.

Plateau's laws describe the shape and configuration of soap films as follows:

1. Soap films are made of entire smooth surfaces.
2. The mean curvature of a portion of a soap film is everywhere constant on any point on the same piece of soap film.
3. Soap films always meet in threes along an edge called a **Plateau border**, and they do so at an angle of $\arccos(-1/2) =$ 120 degrees.
4. These Plateau borders meet in fours at a vertex, and they do so at an angle of $\arccos(-1/3) \approx 109.47$ degrees (the tetrahedral angle).

Configurations other than those of Plateau's laws are unstable and the film will quickly tend to rearrange itself.

Generalisation

The definition of minimal surfaces can be extended to cover constant mean curvature surfaces; surfaces with constant mean curvature, which need not be zero. If a soap film doesn't contain any bubbles, it is called a minimal surface. But if we trap air inside it, and thereby make a bubble, it's no longer a minimal surface. The film now encloses a volume and hence has a nonzero curvature. But, as mentioned, it will still be minimal in the sense that it will present the minimal area that can enclose the given volume; a sphere. We use the term 'minimal constructions' to accommodate all of these.