Modeling radioactive decay with a dice

The process of radioactive decay of isotopes or particles is fundamental to the universe and to particle physics . It is found in lots of places in nature ,anywhere the rate of change of something is proportional to amount of that something

Examples where this phenomena is observed : Radioactive isotopes, bacterial populations ,investments in a bank account.

The other characteristic of radioactive decay is its inherent randomness which leads to some interesting issues in observing and measuring the decay ,problems shared with other inherently random things such as political polling and separately with rolling of dice.

The activity mentioned below use the dice process to give you a tactile ,physical experience with what is really happening in these other processes that are nanoscopic or abstract.

Radioactive Decay

Radioactive decay, also known as **nuclear decay** or **radioactivity**, is the process by which a nucleus of an unstable atom loses energy by emitting particles of ionizing radiation. A material that spontaneously emits this kind of radiation — which includes the emission of energetic alpha particles, beta particles, and gamma rays — is considered **radioactive**.

Radioactive decay is a stochastic (i.e., random) process at the level of single atoms, in that, according to quantum theory, it is impossible to predict when a particular atom will decay.[1] However, the chance that a given atom will decay is constant over time. For a large number of atoms, the decay rate for the collection is computable from the measured decay constants of the nuclides (or equivalently from the half-lifes). A quantity is subject to **exponential decay** if it decreases at a rate proportional to its value. Symbolically, this process can be expressed by the following differential equation, where N is the quantity and λ (lambda) is a positive rate called the **decay constant**:

$$\frac{dN}{dt} = -\lambda N.$$

The solution to this equation (see derivation below) is: Exponential rate of change

$$N(t) = N_0 e^{-\lambda t}.$$

Here $N(t)$ is the quantity at time t, and $N_0 = N(0)$ is the initial quantity, i.e. the quantity at time $t = 0$.

Activity

Roll one hundred dice (multiple times) to represent a sample of hundred radioactive isotopes decaying over time. This does a good job of illustrating the idea of “half life” and the exponential decrease in the activity of a radioactive sample over time. The question is “how many remain after some time.”

Way of doing the dice activity: one hundred at once.

You need: a large supply of dice (about one hundred), a cup or bucket large enough for them, a sheet of paper to record data, and some graph paper for graphing your results.

Imagine: you have a supply of some radioactive isotope that has a 1/6 chance of decaying in the next minute. How much of that isotope remains after six minutes? How about after 20 minutes? How much time does it take for approximately half the sample to decay?

Put all the dice in your cup or jar and roll them on the table.

Separate all the dice that turned up “1”, these are the particles that decayed.

Count and record those that decayed and those that remain, measured after “one minute” passed.

Separate the decayed “ones” into a pile that you can look at later.

Consider: is the pile of decayed dice about as many as you expect? Exactly as many?

Put the others (2-6) back into the cup.

Repeat all these steps and record a measurement of what decayed during and what remains after two minutes. Separate this next batch of dice that turned up “1” into its own pile next to the first – you will save the progression of decays so you can see the process visually, in addition to the numbers you have recorded.

Repeat again for the third minute.

Repeat again and again until they are all decayed.

You now have a sequence of piles you can see (and numbers recorded that you can graph) that represent how many decays happened during each minute. Describe you can see both these seemingly contradictory properties: a) the number of decays is proportional to the number available to decay and b) the number that decayed is random.

Lets represent the situation with a couple graphs. Graph the number that decayed in each minute, using a histogram or bar chart. Separately, graph the number that remain undecayed in the sample, using a histogram or bar chart.

Estimate from your graph, by counting, how much time it takes for approximately half the sample to decay. Draw a mark or an arrow on the horizontal axis of each graph indicating where this time is.

Look up the exponential decay function; if you have a graphing calculator or similar program, plot it with a constant of (0.16666666 = 1/6), in other words, plot: $e^{-x}/_6$. Does that function describe the data you graphed? If you have access to the right software, you might even be able to plot your data and ask the software to fit an exponential, but sketching this one by hand would be good enough. Does the inherent randomness of this process make it difficult to see the exponential nature, and if so, can you think of a change in your procedure

How do you feel about the idea that there was likely a die that remained and didn't decay for fifteen to twenty rolls? Is it possible that one could remain for a hundred or more rolls?

Conclusion

The function that best describes these graphs is a decaying exponential Ne-t/τ, (sometimes also written Ne-λt) where our situation is such that t is time, N is either the initial number of particles that might decay, or the initial activity which is 1/6 of the number of particles, and τ (tau) is the mean lifetime which is (!) 6 seconds or nanoseconds depending on which activity you are doing or λ which is the probability that a particle will decay in the next bit of time. This function occurs a lot for interesting random and non-random situations, but random processes often give rise to behavior that can be described mathematically like this.